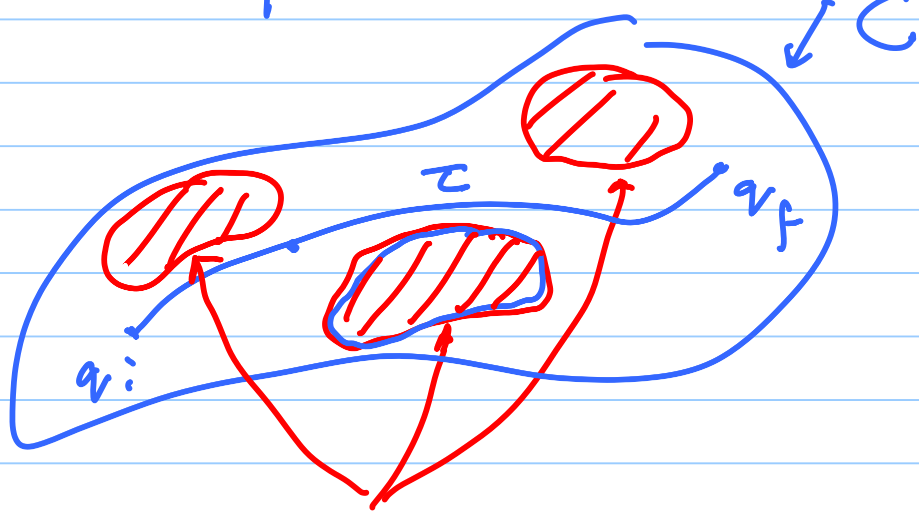
Note Title 1/26/2012

Lecture 7

1) Transform the MP problem to C-space:

formal notion of C-space

Lozano-Perez 1978



$A(\underline{q}) \cap B_i = \phi$

2) the set of configurations that result

in coll. with B_i's

$$CB_i = \{ q : A(q) \cap B_i \neq \phi \}$$

$$CB = \bigcup_i CB_i$$

C-obstacle C_{obs}

$$C_{free} = \{ C - CB \}$$

determine $\tau : [0,1] \longrightarrow C_{free}$ such that

$\tau(0) = q_i$, $\tau(1) = q_f$

Key Issues:

1) C-space is high dimensional for industrial manipulators

mobile robot $\longrightarrow (x, y, \theta)$ 3-dim

6-dof arm $\longrightarrow (\theta_1, \ldots \theta_6)$ 6-dim

Recall ruler folding: alg. complexity is exponential in # of links

2) Complexity of determining CB_i's?

Yes determining the Cobs <u>exactly</u>

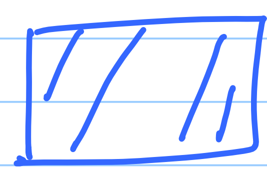
is quite intricate: "non-linear" boundaries

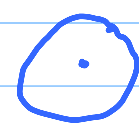
of these Cobs

1) topological structure of CB_i:

a) B is a closed set in CB_i closed?

B

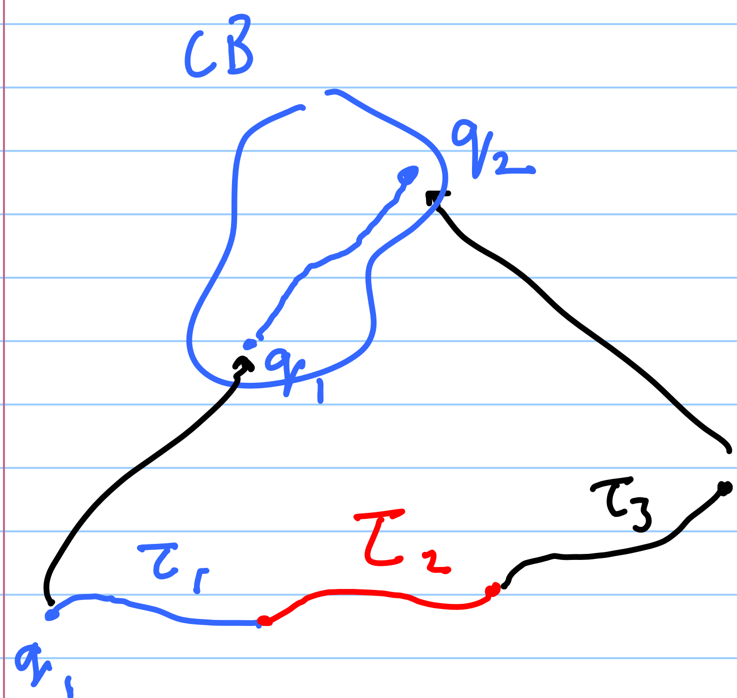


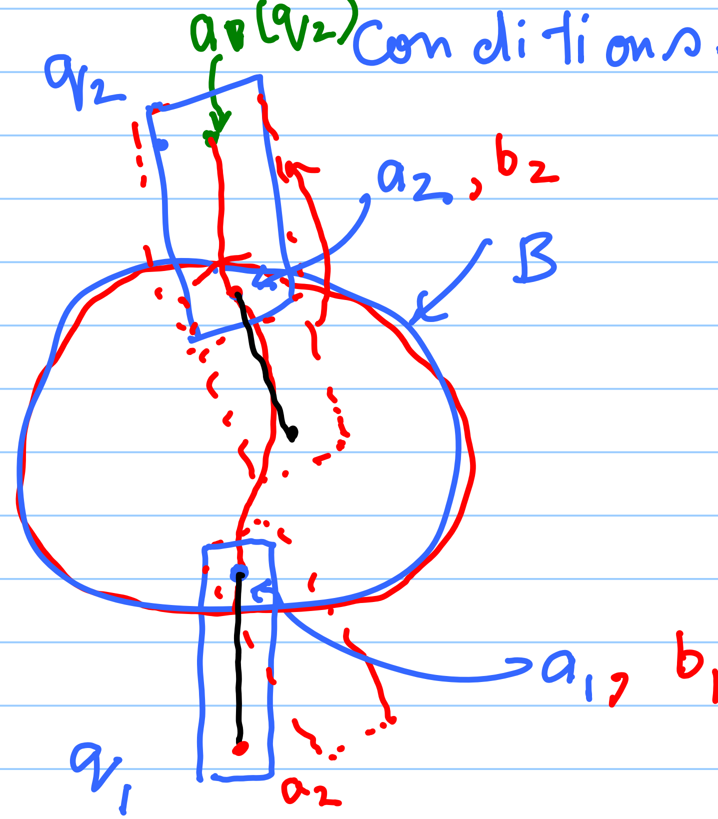


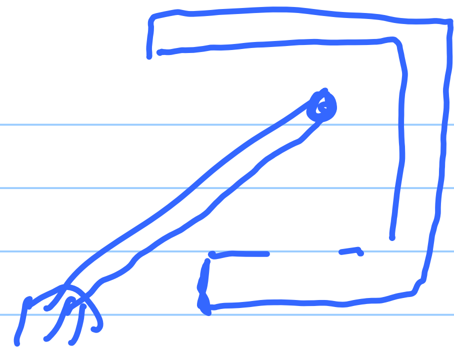
$\Longrightarrow$ Yes

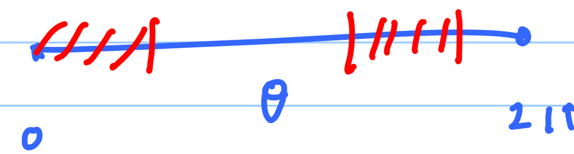
b) B is a connected set. is CB; ?

Yes. under certain conditions.







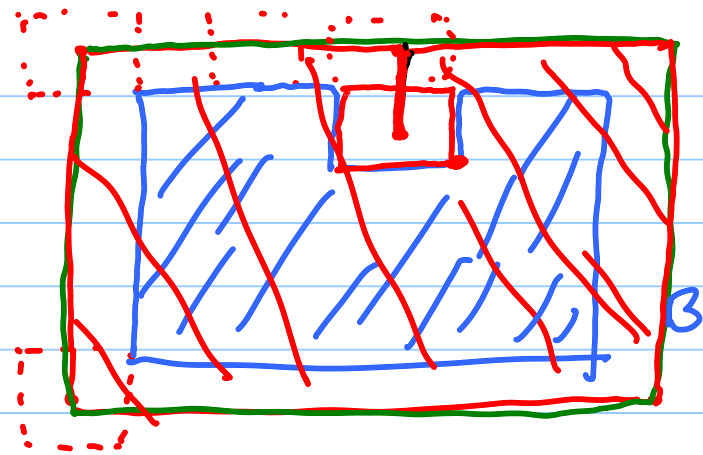


$S = \mathrm{Cl}(\mathrm{Int}(S))$

c) A is regular, B is regular

no dangling edges

CB is regular



$A \to$ red

$C_{obs} \to$ shaded red

$$CB_{contact} = \{ q : A(q) \cap B \neq \phi \wedge \mathrm{Int}(A(q)) \cap \mathrm{Int}(B) = \phi \}$$

$$CB_{overlap} = \{ q : \mathrm{Int}(A(q)) \cap \mathrm{Int}(B) \neq \phi \}$$

$$CB_{contact} = CB - CB_{overlap}$$

$$\boxed{\partial S = cl(S) - Int(S)}$$

$$\partial(CB) \subseteq CB_{contact}$$

$$C_{valid} = C_{free} \cup CB_{contact}$$

semi-free path : $\tau \in C_{valid}$

free " : $\tau \in C_{free}$

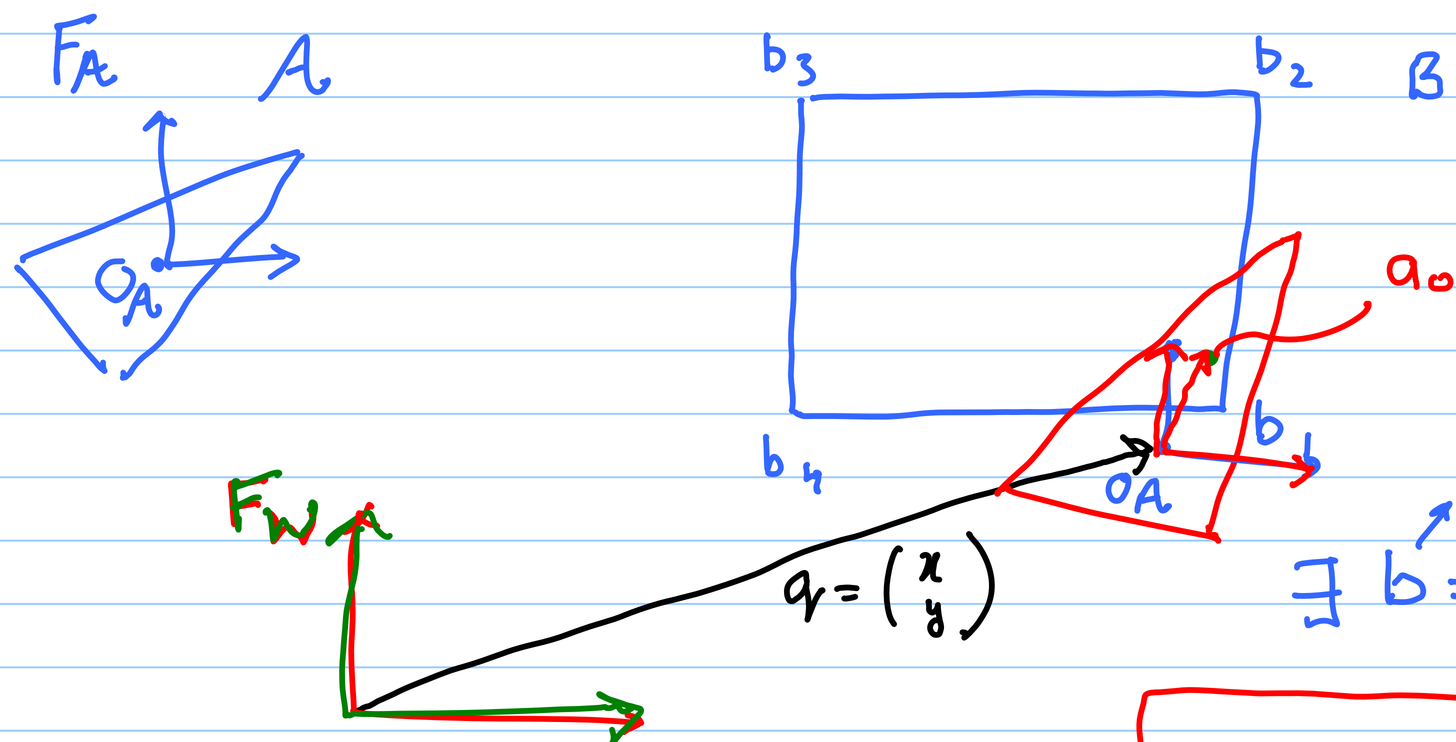
" see Latombe's ch. 2 for more

topological prop. of CB.

Determining CB (geometry) for simple cases:

1) convex polygons (translation only)

2) " (translation + rotation)



$\exists\, b = q + a_0 \in B$

$\boxed{q = b - a_0}$

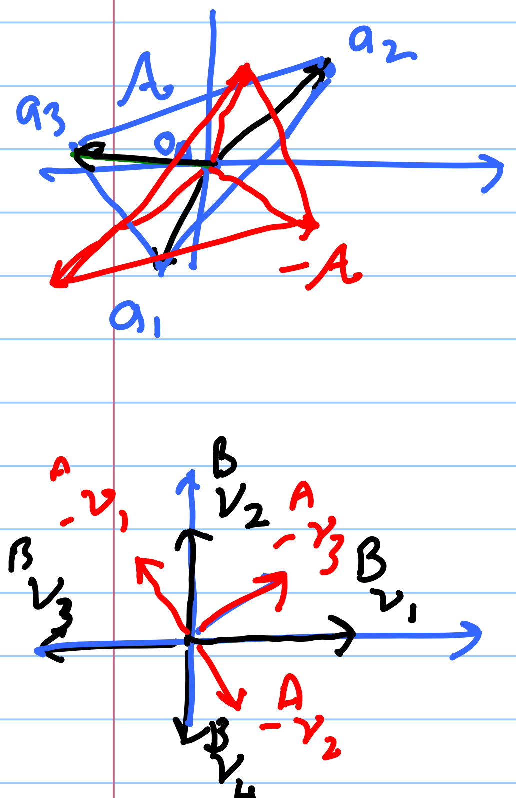
$CB \triangleq \{ q : A(q) \cap B \neq \phi \}$

$$= B \ominus A$$

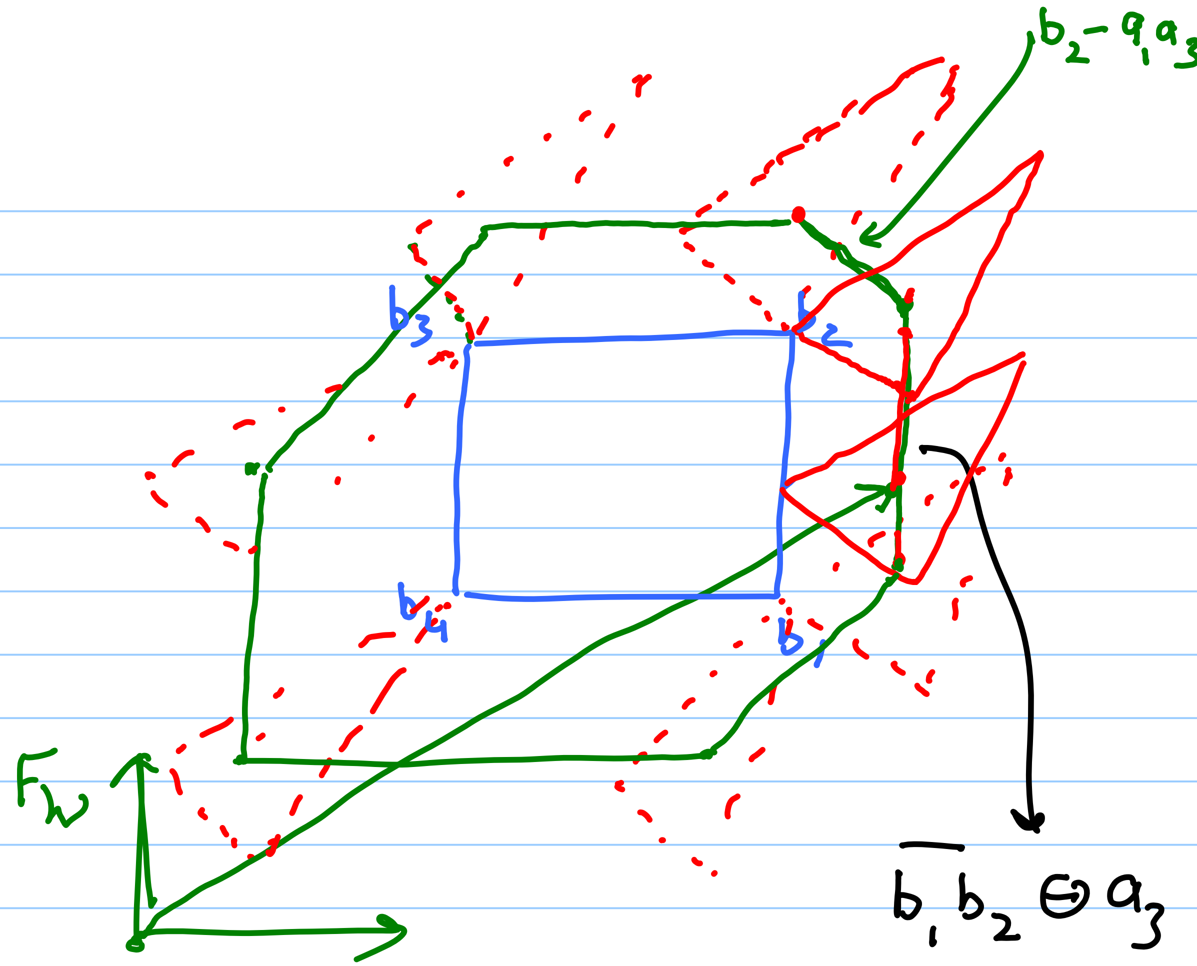
$$S_1 \oplus S_2 = \{ s_1 + s_2 ,\ s_1 \in S_1 ,\ s_2 \in S_2 \}$$

Th. from Geom: A, B are convex sets

$$\partial (A \oplus B) = \partial A \oplus \partial B$$



a_2
a_3
A
o
$-A$
a_1
B
ν_2
A
$-\nu_1$
B
ν_3
A
$-\nu_3$
B
ν_1
A
$-\nu_2$
B
ν_4

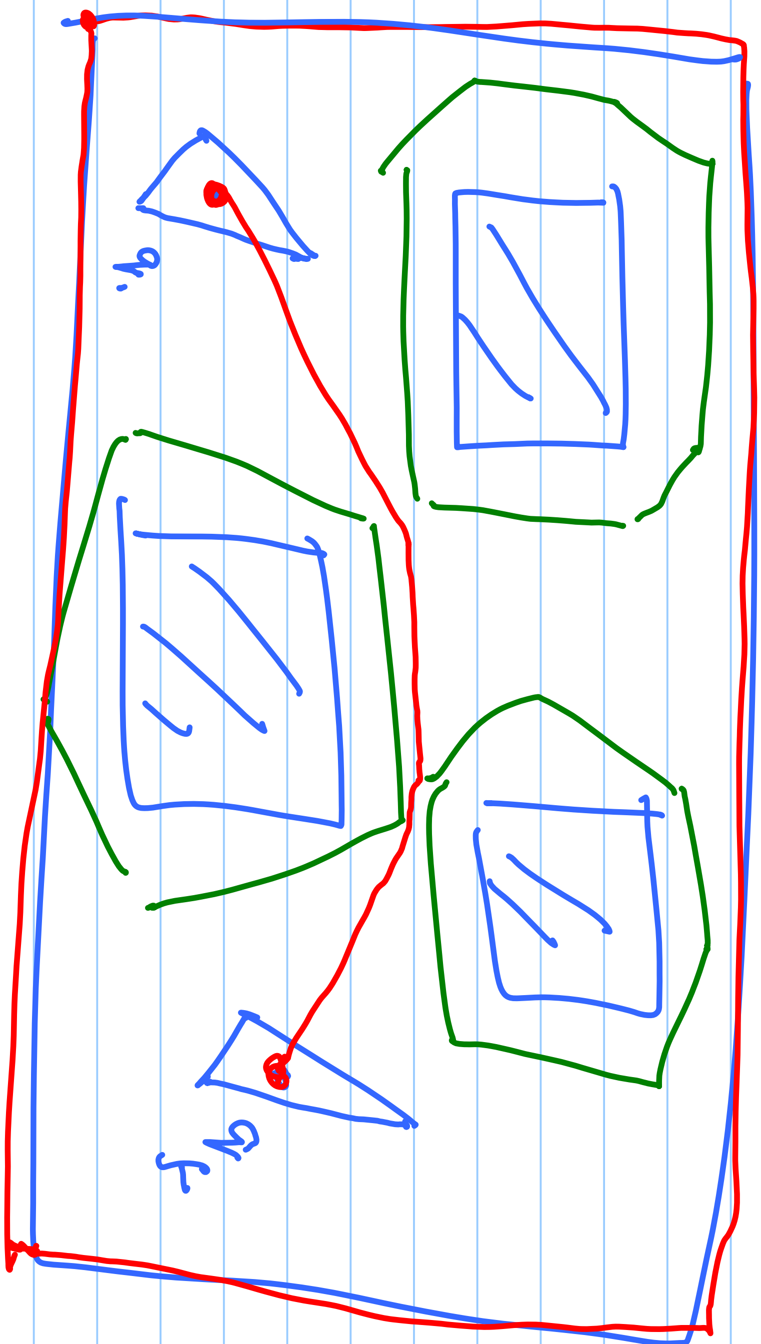


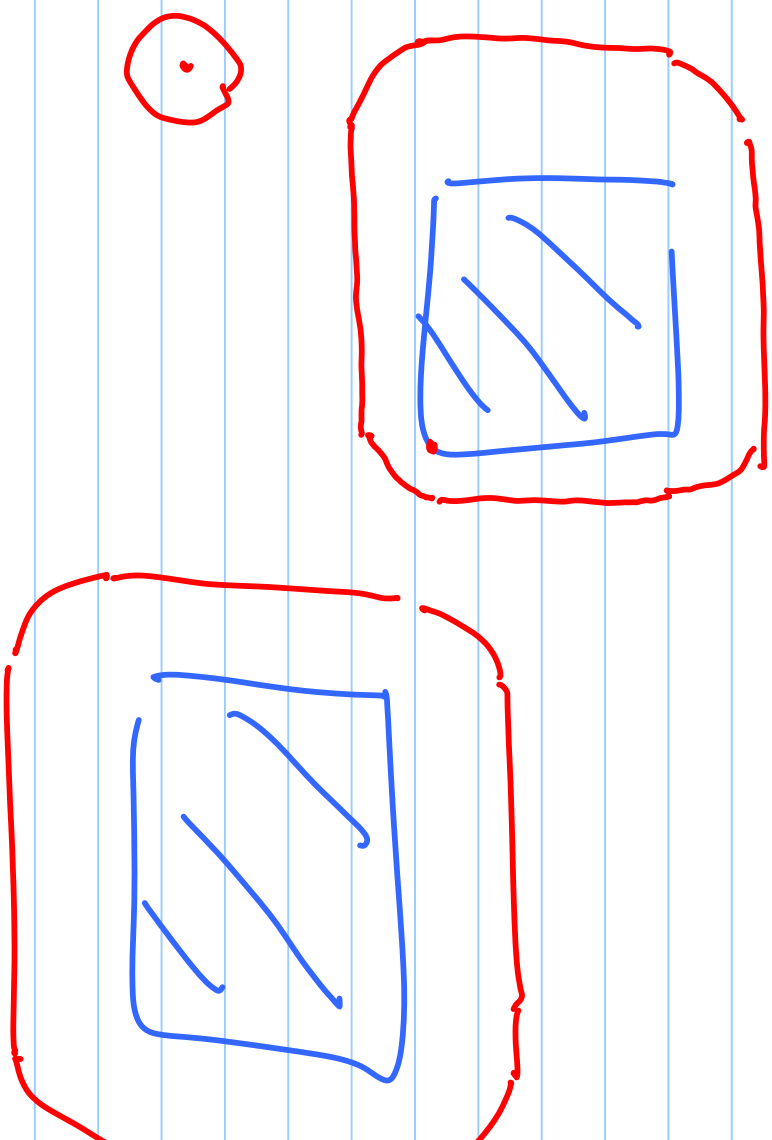
$b_2 - a_1 a_3$
b_3
b_2
b_4
b_1
F_ν
$\overline{b_1 b_2} \ominus a_3$

1) edge of B, vertex of A $\longrightarrow$ type B edge of C_{obs}

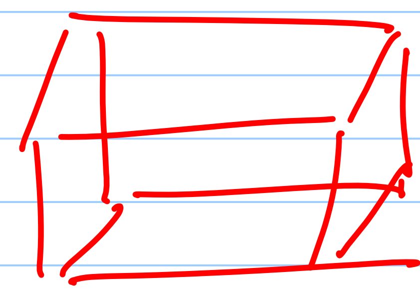
2) vert. of B, edge of A $\longrightarrow$ type A edge

$O(n+m)$ <u>computation</u> to compute C_{obs}





Read Mantyla's 3 chapters on representation of geometric models



(3 types)